The role of executive function skills in the development of children's mathematical competencies

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Abstract

Efforts to understand the domain general skills that are involved in children's mathematics performance have identified that executive functions, the set of skills that control our thoughts and behaviour, play an important role. Previous research has demonstrated that working memory, inhibition and shifting are all related to overall mathematics achievement. More recently, researchers have begun to explore the mechanisms by which executive function skills play a role in mathematics in more detail, by examining the relationship between executive function skills and different components of mathematics, including factual knowledge, procedural skill and conceptual understanding. In this chapter we review the evidence for the role of executive function skills in multiple components of mathematics. We also consider new studies which investigate the precise relationships between executive function skills and arithmetic components across a wide age range.

Keywords: executive functions, working memory, mathematics achievement, factual knowledge, procedural knowledge, conceptual understanding.

Introduction

Proficiency with mathematics is important for success in modern society, and impacts on our health, wealth and quality of life (Gross, Hudson, & Price, 2009; OECD, 2013; Parsons & Bynner, 2005). However, many adults do not have the numeracy skills needed for everyday life and a large proportion of children leave school without achieving the expected level of mathematics skills (Department for Business Innovation & Skills, 2011; Gross et al., 2009). Consequently, researchers have sought to understand the range of cognitive and non-cognitive skills that are involved in mathematics. Over the past two decades, an increasing body of research has identified that executive function, the set of processes that control and guide our thoughts and behaviour, play an important role in mathematics achievement and learning (e.g. review papers by Bull & Lee, 2014; Cragg & Gilmore, 2014; Raghubar, Barnes, & Hecht, 2010).

Executive function is the name given to a group of processes that allow us to respond flexibly to our environment and engage in deliberate, goal-directed, thought and action. Three executive function skills have received the most research attention: *working memory*, the ability to monitor and manipulate information in mind; *inhibition*, the ability to suppress distracting information and inappropriate responses; and *shifting*, the capacity for flexible thinking and switching attention between different tasks.

In this chapter we will review the existing evidence for the role of executive function in mathematics achievement before considering existing and new evidence concerning the involvement of executive function skills in specific components of mathematics. We will finish by presenting recent evidence for the direct and indirect role of executive function on children's mathematics achievement and considering the important distinction between learning and performing mathematics.

The development of executive function

Executive function skills begin to emerge in infancy. By 9 months old most infants show some evidence of attentional control and are able to inhibit unwanted responses and control their behaviour (Diamond, 1985). However, development of executive function skills is not linear, and different sub-skills may emerge at different ages and have different developmental trajectories (Anderson, 2002). For example, the ability to switch flexibly between different tasks does not emerge until 3 or 4 years old (Espy, 1997) and typically develops more slowly than attentional control. Despite the early emergence of some executive functions skills, development is protracted and these skills are among the last cognitive abilities to mature, continuing to develop into late adolescence (Conklin, Luciana, Hooper, & Yarger, 2007; Huizinga, Dolan, & van der Molen, 2006; Luna, Garver, Urban, Lazar, & Sweeney, 2004).

Longitudinal studies indicate that the development of executive function skills is characterised not only by improvements in performance, but also by increasing differentiation in executive sub-skills. While studies with adolescents and adults typically find that executive function is multi-faceted and it is possible to identify separable sub-skills (Huizinga et al., 2006; Miyake et al., 2000), some studies with younger children identify only a single unitary executive factor, or a two-factor model in which inhibition and shifting are undifferentiated (Lee, Bull, & Ho, 2013; Wiebe, Espy, & Charak, 2008).

The protracted and differentiated development of executive function has several implications for understanding how executive function is involved in mathematics. First, age-related differences in the role of executive function skills in mathematics may reflect either changes in the involvement of executive function skills, or changes in the underlying structure of executive function itself. Second, tasks used to assess executive function may draw on different underlying sub-skills at different ages. Finally, during the period of time in which executive function skills develop, the nature of the mathematics activities which children are engaged in also changes dramatically. These factors all combine to add complexity to our understanding of how executive function skills are involved in mathematics, the consequences for interpreting individual differences in mathematics performance and the implications for supporting children's mathematics development.

Executive function and academic achievement

Executive function skills are particularly important when individuals are dealing with novel, rather than routine, situations and activities. This is a key characteristic of learning across all academic subjects and therefore we would expect executive function skills to be an important factor in academic achievement and success in school generally. This is indeed the case, with evidence for the role of executive function skills in mathematics, reading, writing and science outcomes (Best, Miller, & Naglieri, 2011; Nunes, Bryant, Barros, & Sylva, 2012; St Clair-Thompson & Gathercole, 2006). However, over and above the general role of executive function skills in learning and academic achievement generally, it has been suggested that executive function skills are particularly important for mathematics. For example, executive function skills measured at age 5 account for more variance in later mathematics performance than reading (Willoughby, Blair, Wirth, Greenberg, & The Family Life Project Investigators, 2012). Moreover, there is some evidence that the role of executive function skills differs across different academic subjects. In a longitudinal study from kindergarten to Grade 5, the relationship between working memory and mathematics achievement increased, while the relationship between working memory and reading achievement decreased (Geary, 2011). However, other studies have suggested that executive function skills have a domain-general influence on learning and achievement that does not differ across subjects (Best et al., 2011). Differing findings across these studies may reflect the specificity of the tasks selected to measure both executive function skills and academic outcomes. In order to uncover the importance of executive function skills for learning and achievement in mathematics and across the academic spectrum requires sensitive measures, which can pinpoint the specific ways in which executive function skills support learning and achievement.

Executive functions and mathematics achievement

A wealth of evidence, largely from correlational and longitudinal studies, has demonstrated a general relationship between executive function skills and overall mathematics achievement. These studies have shown that scores on cognitive tests of executive function skills are associated with concurrent or future mathematics performance, as measured by standardized or curriculum-based mathematics tests. Such studies have largely focused on the role of working memory, however inhibition and shifting have received increased attention in the past few years.

Working memory and mathematics achievement

The majority of studies which have explored the involvement of working memory in mathematics achievement have been based upon the Baddelely and Hitch (1974) model of working memory, whereby working memory is made up of short-term stores for verbal and visuospatial information, coordinated by a central executive that allows the manipulation and storage of information at the same time. Drawing on this model, researchers have demonstrated that working memory capacity is a strong predictor of current and future mathematics achievement (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Fuchs et al., 2010; Hecht, Torgesen, Wagner, & Rashotte, 2001; Peng, Namkung, Barnes, & Sun, 2016).

Beyond a general association between working memory and mathematics achievement, researchers have attempted to pinpoint which functions of working memory are most critical for explaining variance in mathematics achievement (e.g. see meta-analysis by Friso-van den Bos et al., 2013). This has revealed that tasks which require the simultaneous storage and manipulation of information (executive working memory) show a stronger relationship with mathematics achievement than tasks which simply measure the short-term storage of information, particularly in relation to the storage of verbal information.

Several studies have investigated whether there is a stronger relationship between verbal or visuospatial working memory and mathematics achievement. This has produced mixed findings with some studies suggesting that verbal working memory plays a greater role in mathematics achievement (e.g. Bayliss, Jarrold, Gunn, & Baddeley, 2003; Friso-van den Bos et al., 2013) while others find that visuospatial working memory is more important (e.g. Schuchardt, Maehler, & Hasselhorn, 2008; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014). A recent meta-anaysis of 110 studies found no difference in the strength of the relationship between mathematics achievement and verbal working memory, visuo-spatial working memory or numerical working memory (Peng et al., 2016). One explanation for these conflicting findings is that the importance of verbal vs. visuospatial working memory changes over development. Li and Geary (2013) found that verbal working memory predicted mathematics achievement in 7-year-olds, but that gains in visuospatial short-term memory predicted mathematics achievement at 11 years old. These changes could reflect either developmental changes in the involvement of working memory, or differences in the nature of mathematical activity (e.g. a shift from a focus on arithmetic to more advanced mathematical topics) across these years. However, few studies have explored the relationship between comprehensive measures of verbal and visuospatial working memory and mathematics achievement across development.

In a recent study, we explored whether there was a stronger relationship between verbal or visuospatial working memory and mathematics achievement in a sample of children across a wide age range (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017). Groups of children aged 8-9 years, 11-12 years, 13-14 years and young adults completed a large battery of executive function tests, including measures of verbal and visuospatial short-term and working memory. Verbal working memory was measured via a sentence span task. Participants heard a sentence with the final word missing and provided an appropriate word. After responding to a series of sentences they were asked to recall the final word of each sentence in the series, in the correct order. Participants also completed the storage and processing elements separately. Visuospatial working memory was assessed via a complex span task. Participants saw a series of 3 x 3 grids each containing three symbols and they had to point to the 'odd-one-out' symbol that differed from the other two. After responding to a series of grids participants were asked to recall the position of the odd-one-out on each grid in the series, in the correct order. Again, participants also completed the storage and processing elements separately. Mathematics achievement was assessed using the Mathematics Reasoning subtest of the Wechsler Individual Achievement Test. Both verbal and visuospatial working memory performance were significant unique predictors of mathematics achievement (see Figure 1), and these relationships were consistent across age groups. This suggests that the contribution of verbal and visuospatial working memory may be very similar and consistent across development, at least from mid-childhood to adulthood.

As well as the distinction between verbal and visuospatial working memory, a small number of studies have attempted to identify the exact components of working memory that contribute to mathematics achievement. Working memory measures inevitably also require the short-term storage and processing of information. When separate measures of short-term storage and processing are used alongside a complex working memory span task, in other words, when the processing component of a complex span task (e.g. sentence completion) is also measured alone, without the memory demands, a variance partitioning approach can be used to isolate the variance in performance that is associated with each element of working memory. This approach was taken in two studies by Bayliss and colleagues (Bayliss, Jarrold, Baddeley, Gunn, & Leigh, 2005; Bayliss et al., 2003) who found that all components of working memory play some role in mathematics achievement, but the combined storage and processing of verbal information is particularly important, at least in childhood.

We also used a variance partitioning approach to explore which specific components of working memory are important for mathematics achievement (Cragg et al., 2017). We found that for verbal information, short-term memory, working memory and the shared variance between short-term and working memory all accounted for significant variance in mathematics achievement. For visuospatial information there was an additional role for processing, such that short-term memory, processing, working memory, shared variance between short-term and working memory, and shared variance between short-term, processing and working memory all accounted for significant variance in mathematics achievement. In both cases it was the shared variance between short-term and working memory that accounted for the greatest variance in mathematics achievement. This suggests that the ability to simply hold information in mind is as important for mathematics achievement as the ability to hold information while processing.

Inhibition, shifting and mathematics achievement

The role of inhibition and shifting in mathematics achievement has received less attention than working memory, although recent studies have increasingly begun to focus on these skills. The evidence is more inconsistent, with some studies finding a significant relationship between inhibition or shifting with mathematics performance in preschool and school-aged samples (Blair & Razza, 2007; Clark, Pritchard, & Woodward, 2010; Merkley, Thompson, & Scerif, 2016; St Clair-Thompson & Gathercole, 2006; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013; Yeniad, Malda, Mesman, van IJzendoorn, & Pieper, 2013) while others find no relationship (Lee et al., 2012; Monette, Bigras, & Guay, 2011; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). This may be explained, in part, by shared variance with other cognitive skills. There is some evidence that inhibition is only a significant predictor of mathematical performance when shifting skills are not taken into account (Bull & Scerif, 2001; Van der Ven et al., 2012). Similarly, inhibition and shifting may be significant predictors of mathematics performance when considered alone, but do not make a unique contribution when working memory (Lee et al., 2012) or intelligence (Yeniad et al., 2013) are taken into account. Although Espy et al. (2004) found that inhibition was a significant predictor of mathematics after controlling for both working memory and shifting skills.

Another explanation for this inconsistent evidence, particularly concerning inhibition, is the nature of the executive function tasks. Different types of inhibition tasks tap into varying aspects of inhibition skill (e.g. response inhibition vs. interference control). These different forms of inhibition may not represent a single underlying construct, and indeed show different developmental trajectories (Huizinga et al., 2006) and therefore may have different involvement in mathematics. Consequently, evidence for the relationship between inhibition and mathematics may depend on the specific forms of inhibition and tasks selected.

Further questioning the idea of a single, domain-general inhibitory system (Egner, 2008), there is evidence for domain-specificity in the relationship with mathematics. There is some evidence that the relationship between inhibition and mathematics achievement is stronger when the inhibition task involves numerical rather than non-numerical stimuli. Several studies have found a significant relationship between children's mathematics achievement and performance on a number-quantity stroop task but no relationship between mathematics and colour-word stroop performance (Bull & Scerif, 2001; Navarro et al., 2011; Szucs et al., 2013) although other studies did not find this pattern (De Weerdt, Desoete, & Roeyers, 2013). We found some evidence for domain-specific effects: numerical inhibition (selecting the more numerous of two dot arrays ignoring the size of the dots), but not non-numerical inhibition (selecting the larger animal in real life ignoring the size of the animals on the screen) was a significant predictor of mathematics achievement in children aged 8 – 14 years old and adults (Cragg et al., 2017), However, further research is needed to explore domain-specific effects for inhibition, and in particular the extent to which this depends on the nature of the mathematics task.

A final explanation for the inconclusive evidence regarding the involvement of inhibition and shifting in children's mathematics performance is the nature of the mathematical tasks. It is difficult to pinpoint the role of inhibition and shifting in learning or performing mathematics when mathematics outcomes are measured with a general standardized or curriculum measure of mathematics. Instead, the role for these executive function skills is likely to be more specific and differ according to the mathematical activity. As we see below, it is therefore important to consider multiple components of mathematics in order to understand how and when executive function skills are involved.

Multiple components of mathematics

It is well established that mathematics is a complex, multi-componential skill. Not only are there multiple domains, such as arithmetic, algebra, geometry, and statistics, but each of these domains involves multiple skills. For example, when learning arithmetic children need to learn number symbols and facts, become proficient with different operations (addition, subtraction etc.), understand and apply underlying concepts and principles, develop problem solving approaches and be able to apply arithmetic to real world situations. These different components of arithmetic are not hierarchically ordered, instead there may be complex relationships among these individual skills (Dowker, 2005). Studies have shown that children may have strengths in one area despite weaknesses in another. For example, children may understand arithmetical concepts despite difficulties in performing calculations or vice-versa (Canobi, 2004; Gilmore & Papadatou-Pastou, 2009).

A recent meta-analysis has explored the role of working memory across different domains of mathematics (Peng et al., 2016). This identified that working memory is most strongly related to whole-number calculations and word-problem solving, and has weaker relationship with geometry. However, this study only considered different domains and it is crucial to take into account different component skills when we try to identify the role of executive function skills. This is particularly true if we want to go beyond simply identifying correlations to understand precisely how executive function skills are involved in mathematics learning and performance; the role of working memory, inhibition and shifting is likely to differ for different components. Researchers have put forward many suggestions for the ways in which different executive function skills support mathematics, for example working memory may be important for holding interim solutions in mind during computation, inhibition may be needed to suppress unwanted number facts during retrieval, and shifting may be involved when switching between different operations and number representations (Bull & Lee, 2014; Cragg & Gilmore, 2014). However, evidence for these specific mechanisms cannot be obtained from correlational studies that use general curriculum or standardized measures of overall mathematics achievement. These measures draw on a wide variety of mathematics skills, the precise constellation of which may differ from test to test, across ages even with the same test, and even from individual to individual, depending on the strategies they use. For example, a set of arithmetic problems may be solved by some children via computational strategies, but by other children via retrieval. It is likely that the role of executive function skills will differ according to the strategy used, and thus the overall relationship between performance on the test and executive function performance may not be informative of the precise involvement of executive function skills. Consequently, we need to investigate how executive function skills support the learning and performance of different mathematical skills, and where possible to go beyond correlational designs to use experimental techniques which directly implicate specific executive function skills in specific mathematical processes.

Below we review existing and new evidence for the role of executive function skills in specific components of mathematics. We focus on procedural, factual and conceptual knowledge of arithmetic as these are the skills that have received the most attention to date.

Executive function skills and components of arithmetic

Executive function skills and procedural skill

Procedural knowledge of arithmetic has been defined as the ability to perform an ordered sequence of steps to solve a problem or knowing "how to" (Baroody, 2003; Hiebert & Lefevre, 1986). This involves accurately and efficiently selecting and performing appropriate operations. Executive function skills are likely to be important for procedural knowledge to represent the question, store interim solutions or keep track of counts, to select the appropriate strategy and inhibit inappropriate ones, and to switch between operations, strategies and notations. Evidence for the role of executive function skills and procedural skill comes largely from two types of studies: correlational or longitudinal battery studies which focus on specific measures of procedural skill, rather than general standardized measures of achievement; and dual-task studies which explore the online involvement of working memory while arithmetic problems are solved with procedural strategies. Evidence from these two sources will be considered in turn below.

Correlational studies have found a relationship between working memory and a variety of measures of procedural skill. Cowan and Powell (2014) found that a composite measure of domain-general skills, including working memory measures, was significantly related to basic calculation fluency, written arithmetic and word problem solving. Fuchs et al. (2010) identified that central executive processes in particular were important for predicting development in multi-digit arithmetic performance. Hecht, Close and Santisi (2003) found that verbal working memory was related to fraction computation (but not conceptual understanding of fractions). Inhibition has also been implicated in procedural skills, with evidence that children with better inhibitory control made more use of the most efficient strategy to solve arithmetic problems (Lemaire & Lecacheur, 2011). Similarly, children's performance on a measure of task switching was related to procedural skills including basic calculation and word problem solving (Andersson, 2010).

There is some evidence that executive functions may play a larger role in procedural skills for younger children than older children (Best et al., 2011; Friso-van den Bos et al., 2013), however this comes largely from studies which use general measures of mathematics achievement, rather than specific measures of procedural skill. Over development and schooling children's procedural skills become more automatic and they make use of different strategies. This may reduce the demands on executive function skills. However, few studies have explored the role of executive function skills on specific measures of procedural skills across development. We explored the relationship between working memory, inhibition and shifting and procedural skill in a wide age range (Cragg et al., 2017). Participants aged 8-9 years, 11-12 years, 13-14 years and young adults completed a battery of executive function measures. Procedural skills were measured by response times to solve a set of single and double-digit arithmetic problems. The problems varied across age groups to ensure that all groups would solve them via procedural strategies, rather than retrieval or guessing. The younger two age groups solved addition and subtraction problems, while the older two age groups solved addition, subtraction, multiplication and division problems. Verbal working memory, visuospatial working memory and inhibition of numerical information were all significant independent predictors of procedural skill and these relationships did not interact with age group. Set shifting was not related to procedural skill when entered into the same model (see Figure 1). The specific components of working memory that were important for procedural skill were further investigated via a variance partitioning approach (Bayliss et al., 2003). This revealed that verbal and visuospatial working memory, and the shared variance between short-term memory and working memory, as well as visuospatial short-term memory accounted for significant variance in procedural skills. Overall the proportion of variance in procedural skills (15%) accounted for by executive function skills was less than that for overall mathematics achievement (34%).

An alternative approach to exploring the role of working memory on procedural skills is via dual-task studies. In this experimental approach, participants solve arithmetic problems both with and without a concurrent working memory load. The impact of the working memory load on performance on the arithmetic problems can shed light as to the involvement of working memory for calculation. Several studies using this methodology have identified the impact of working memory load on adults' procedural arithmetic skills. These studies have revealed that there is more interference in arithmetic performance from secondary tasks requiring central executive involvement, rather than the simple storage of information (Imbo & Vandierendonck, 2007b; Rammelaere, Stuyven, & Vandierendonck, 1999). When exploring the impact of concurrent working memory load on arithmetic procedural skills it is important to take account of the strategies used to solve the problems. Adults and children can use a variety of strategies to solve arithmetic problems, including *retrieval* from long-term memory, or procedural strategies such as *decomposition* (breaking down the problem into simpler problems e.g. 5 + 7 could be solved by 5 + 5 = 10, 10 + 2 = 12) or *counting*. It is essential to take account of the strategies that participants use to solve arithmetic problems in a dual-task paradigm, because the involvement of working memory is likely to differ according to the strategy used. Studies have explored this by controlling the strategy which adults are required to use to solve arithmetic problems. This approach has identified that working memory tasks with a central executive component interfere with procedural strategies including counting and decomposition (Imbo & Vandierendonck, 2007b) and there is some evidence that counting strategies suffer greater impairment (Hubber, Gilmore, & Cragg, 2014).

Few studies have explored the impact of working memory load on children's procedural skills, or how the effects of working memory load change across development. However, it is plausible that children rely on working memory to a greater extent when solving arithmetic problems compared to adults through the use of less efficient strategies. Imbo and Vandierendonck (2007a) explored the impact of a working memory secondary task (continuous choice reaction time task) when children aged 10 – 12 years solved single digit addition problems by either counting, retrieval or decomposition. They found that working memory load had an impact on both procedural strategies, which was greater for decomposition. However, this study did not explore whether the effects of working memory load differ across development.

We have recently explored the impact of dual-task load on arithmetic problem solving across a wide age range (Cragg, Hubber, Keeble, Richardson, & Gilmore, under review). Children aged 9 – 11 years, 12 – 14 years, and adults solved addition problems using retrieval, decomposition and counting under three conditions: alone (no secondary task), with a control load (with a secondary task that did not require working memory load) and with a working memory load (with a secondary task that required working memory load). The working memory load task was a n-back task involving the monitoring of either verbal or visuospatial information. The control load task involved the same responses as the working memory load task, but without the need to monitor and update information. This study revealed that working memory load interfered with the performance of arithmetic whether participants solved problems via counting, decomposition or retrieval. Surprisingly, the effect of working memory load was the same for children and adults. This suggests that the processes involved in the online performance of arithmetic are similar across development. There was some evidence that monitoring and updating verbal information interfered more than visuospatial information when counting, but not when using retrieval or decomposition strategies. However, overall concurrent visuospatial load (with or without an associated working memory load) slowed arithmetic performance to a greater extent than verbal load. This study demonstrated the importance of working memory for solving arithmetic problems using procedural strategies.

Executive function skills and factual knowledge

The ability to retrieve the solutions to simple arithmetic problems is a focus of early mathematics instruction and an important indicator of overall mathematical performance (Geary, 2004). Several models have been proposed to account for the storage and retrieval of number fact knowledge, many of which are based on an associative network. For example, Siegler and colleagues proposed the *Distributions of Associations* network (Lemaire & Siegler, 1995; Siegler, 1988). According to this model, children initially solve problems using procedural strategies. Every time a problem is solved in this way, the association between the operands and the results is strengthened. With practice the operands become more strongly associated with the correct answer. Once the association between the operands and answer exceeds a threshold then the answer can be retrieved. This type of model can account for the types of errors typically seen when children and adults retrieve the answers to arithmetic problems. Incorrect answers are more often related to the operands than unrelated. For example, the answers to addition problems may be retrieved instead of multiplication problems (e.g. answering 30 when asked to retrieve the answer to 5 + 6), or incorrect answers might be associated with one of the operands (e.g. answering 21 when asked to retrieve the answer to 7 × 4).

Based on this type of model of number fact retrieval, researchers have begun to consider the role of executive functions in the storage and retrieval of factual knowledge. Most attention has been paid to working memory and inhibition. It has been proposed that one role of working memory is to activate information in long-term memory (e.g. Cowan, 1999) and consequently it might be expected that working memory capacity is associated with fact retrieval. Evidence for this comes from both correlational and experimental studies.

Correlational studies have demonstrated that children with low working memory capacity are less accurate when retrieving solutions (Andersson, 2010; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007) and choose to make less use of retrieval than their peers with greater working memory capacity (Geary, Hoard, & Nugent, 2012). Our recent work found that both verbal and visuospatial working memory capacity were significant independent predictors of factual knowledge (correct responses to simple arithmetic problems within 3 seconds). This relationship was stable from age 8 to adults (Cragg et al., 2017). This association was further investigated using a variance partitioning approach to identify the specific components of working memory that were associated with factual knowledge. This revealed that for verbal information, short-term memory, working memory and the shared variance between short-term and working memory all accounted for significant variance in factual knowledge. For visuospatial information there was a greater role for processing, such that short-term memory, processing, working memory, the shared variance between short-term memory and working memory, and the shared variance between short-term memory, processing and working memory all accounted for significant variance in factual knowledge. In total, visuospatial working memory accounted for greater variance in factual knowledge than verbal working memory. This is somewhat surprising given that number facts are thought to be stored in a verbal code. It is possible that this reflects the role of visuospatial working memory in learning, rather than storing or retrieving, number facts.

Alongside evidence from these correlational studies, experimental dual-task studies have also demonstrated a role for working memory in fact retrieval. Concurrent working memory load has been found to interfere with the retrieval of arithmetic number facts by adults (Hubber et al., 2014; Imbo & Vandierendonck, 2007b), although the interference may be less than that for procedural strategies. Similarly, for children, working memory load interferes with the retrieval of number facts in comparison to either no load (Imbo & Vandierendonck, 2007a) or to a control load (Cragg et al., under review). Put together with the correlational findings, these studies suggest that working memory is involved not only in learning number facts but also in the online storage and retrieval of facts when children are actively problem solving.

As well as working memory, there has been increasing interest in the role of inhibition in number fact knowledge. It has been proposed that inhibition is required to suppress the retrieval of incorrect solutions (Bull & Lee, 2014; Cragg & Gilmore, 2014). However, evidence in support of this is still somewhat scarce. Most evidence from correlational studies for the role of inhibition in specific arithmetic skills has focused on procedural skills (e.g. Lan, Legare, Ponitz, Li, & Morrison, 2011). Some evidence for the role of inhibition is starting to emerge from experimental paradigms. For example, De Visscher and Noël (2014) used an interference paradigm to explore children's susceptibility to interference in memory, and found that children with poor knowledge of multiplication facts were more sensitive to interference in memory.

We explored the relationship between inhibition and factual knowledge in children aged 8 to 14 and young adults (Cragg et al., 2017). Factual knowledge was measured as the proportion of correct responses given to simple arithmetic problems within 3 seconds. Inhibition skill was measured with two tasks – one which involved the inhibition of numerical information and one which involved the inhibition of non-numerical information. This revealed that inhibition of numerical information was a unique predictor of factual knowledge, over and above working memory and shifting, and that this relationship was consistent across development.

Executive functions and conceptual understanding

Conceptual knowledge has been defined as understanding of the principles and relationships that underlie a domain (Hiebert & Lefevre, 1986) or *knowing why* (Baroody, 2003). It is widely recognised that good conceptual understanding is important for success in mathematics (see review by Rittle-Johnson & Schneider, 2015). Although many studies have investigated the development of conceptual understanding, little attention has been paid to the role of domain-general skills, and particularly executive functions, in conceptual understanding. However, it is plausible that executive functions play a role in both the acquisition of conceptual knowledge, and the selection of conceptually-based strategies. Inhibition and shifting may be involved in suppressing a prepotent procedural strategy and switching attention to identify underlying conceptual relationships. Working memory may be required to activate conceptual knowledge in long-term memory (e.g. Cowan, 1999).

The few studies which have explored the role of executive functions in conceptual understanding have produced a mixed picture. Robinson & Dubé (2013) found that children with good inhibitory control were more likely to make use of an alternative conceptually-based strategy rather than computation to solve problems than children with poor inhibitory control. Similarly, Watchorn et al. (2014) found that children's conceptual understanding was associated with good attentional skills, at least for children who also had good procedural skills. Andersson (2010) found that visuospatial working memory, but not shifting, was a predictor of later conceptual understanding. However, other studies exploring the role of working memory in conceptual understanding have failed to find a relationship, at least within the domain of fractions (Hecht et al., 2003; Jordan et al., 2013).

We explored the relationship between verbal and visuospatial working memory, inhibition and shifting with conceptual understanding in children aged 8 – 14 and young adults. Conceptual understanding was measured using a task in which participants were asked to identify conceptual relationships (e.g. inversion, commutativity, associativity) between pairs of problems (based on Canobi, 2004). Across all age groups, only verbal working memory was significantly associated with conceptual understanding (see Figure 1). We further explored this using a variance partitioning approach and identified that verbal working memory and the shared variance between verbal short-term and working memory were (marginally) significant predictors of conceptual understanding. Visuospatial processing (but not storage) also explained significant variance in conceptual understanding. Overall, only 5% of variance in conceptual understanding was accounted for by executive function skills (compared with 15% of procedural skills and 34% of mathematics achievement).

Overall, the role of executive functions in conceptual understanding remains unclear. One explanation for this is that it is difficult to measure conceptual understanding in isolation. Further research is needed to uncover the domain-general skills involved in both acquiring conceptual understanding and using it in problem-solving. A clearer picture of the skills involved might help to explain why some children struggle to develop conceptual understanding, while other children have better conceptual understanding than expected given their procedural skills (Gilmore & Papadatou-Pastou, 2009; Watchorn et al., 2014).

Direct and indirect influences of executive functions on mathematics achievement

As described above, there is a large body of evidence for the involvement of executive function skills in overall mathematics achievement, as well as some evidence for the role of executive function skills in specific components of mathematics: procedural skill, factual knowledge and conceptual understanding. Hierarchical models of mathematics (Fuchs et al., 2010; Geary, 2004; LeFevre et al., 2010) propose that the role of executive function skills in specific components of mathematics accounts for the role of these skills in overall mathematics achievement. For example, Geary (2004) put forward a hierarchical model in which basic cognitive systems, including executive function skills, underlie conceptual and procedural knowledge of a domain which in turn support overall competence in that domain. In other words, the role of executive function skills in conceptual and procedural knowledge fully accounts for the role of executive function skills in overall achievement. This mediating role of conceptual and procedural knowledge in explaining the relationship between executive function skills and mathematics achievement has not previously been tested.

We tested this relationship by identifying the direct and indirect effects of executive function skills on mathematics achievement in children aged 8 – 14 years and young adults (Cragg et al., 2017). We first explored the relationship between procedural, conceptual and factual knowledge and mathematics achievement using hierarchical regression. All three mathematical components were significant independent predictors of mathematics achievement, and this pattern remained stable across age. We next explored the relationship between executive function skills and mathematical components. As described in more detail in the sections above, we found that verbal and visuospatial working memory and numerical inhibition were significant independent predictors of factual knowledge and procedural skill, and that verbal working memory was a significant predictor of conceptual understanding. Non-numerical inhibition and shifting were not significantly related to any outcomes. Again, this pattern was stable across age groups. Finally, to test the hierarchical model, we performed mediation analyses to test whether factual knowledge, procedural skill and conceptual understanding mediated the relationship between working memory and mathematics achievement. Inhibition was not included in this model because it was not significantly associated with mathematics achievement. Two separate models were tested, for verbal and visuospatial working memory separately.

This analysis revealed significant indirect effects of verbal working memory on mathematics achievement via all three component skills: factual knowledge, procedural skill and conceptual understanding. There were also significant indirect effects of visuospatial working memory on mathematics achievement via factual knowledge and procedural skill but not conceptual understanding. In both models, however, there remained a substantial direct effect of working memory on mathematics achievement. These analyses indicate that although factual knowledge, procedural skills (and conceptual understanding) partially mediate the relationship between executive function skills and mathematics achievement, there remains a substantial role for executive function skills that remains to be explained. This direct path may represent the role of working memory in problem solving; for example to identify the mathematical problem, construct a problem representation and select a strategy. Alternatively, this direct path may represent the role of working memory more generally in learning, which may be better captured by performance on mathematics achievement tests than on measures of specific components of arithmetic. Further work is required to better understand this as yet unexplained role played by working memory in mathematics.

Conclusions: executive functions and learning vs. performance

A large body of evidence now exists to demonstrate that executive function skills play a role in children's learning of mathematics. Over the last decade researchers have begun to go beyond broad general relationships to understand at a more nuanced level the precise role that different executive function skills play in different components of mathematics. This endeavour has benefitted from multiple sources of evidence, including experimental as well as correlational studies to pinpoint mechanisms and causal relationships. With regard to children's procedural and factual knowledge substantial progress has been made in understanding the role of executive function skills, however further work is needed in regard to conceptual understanding.

One important distinction which requires further attention is that between performance of mathematics and learning mathematics. It is likely that executive function skills play a different role when children are learning new mathematical material compared with performing known mathematical procedures. Many of the methods currently being used to explore the role of executive functions are unable to distinguish these factors. For example, a correlation between working memory capacity and knowledge of number facts could indicate that working memory is required to store and retrieve number facts, or could be a legacy from the involvement of working memory in learning these number facts in the first place. Experimental methods, for example dual task studies, can help to isolate the role of executive functions in the performance of mathematics. However, to reveal the role of executive functions in learning will require the use of different techniques. These might include investigating individual differences in children's ability to learn new mathematics material in school, or alternatively studying learning more directly in lab-based studies (e.g. via artificial learning paradigms).

A thorough understanding of the role of executive functions in mathematics learning and performance can help us to make sense of the wide individual differences in children's success with mathematics. This in turn can help to understand why some children struggle with mathematics, identify those who are at risk of developing difficulties, and develop teaching approaches or interventions which may benefit all children. Current attempts at improving mathematics outcomes via training executive function skills have failed to show successful transfer (Melby-Lervåg & Hulme, 2013). This may be because these approaches fail to take account of the specific ways in which executive functions are involved in mathematics. A more profitable avenue to explore may be to understand and manage the executive function demands of classroom activities. As described above some progress has been made in this regard, but further research, based on detailed models of mathematical cognition, is required.

Figure Caption

Figure 1: Variance in mathematics achievement, factual knowledge, procedural skill and conceptual understanding explained by executive function skills. Executive function skills were verbal working memory (WM), visuospatial (VS) working memory, inhibition of non-numerical information, inhibition of numerical information and shifting. Adapted from (Cragg et al., 2017). * $p < .05$, ** $p < .01$

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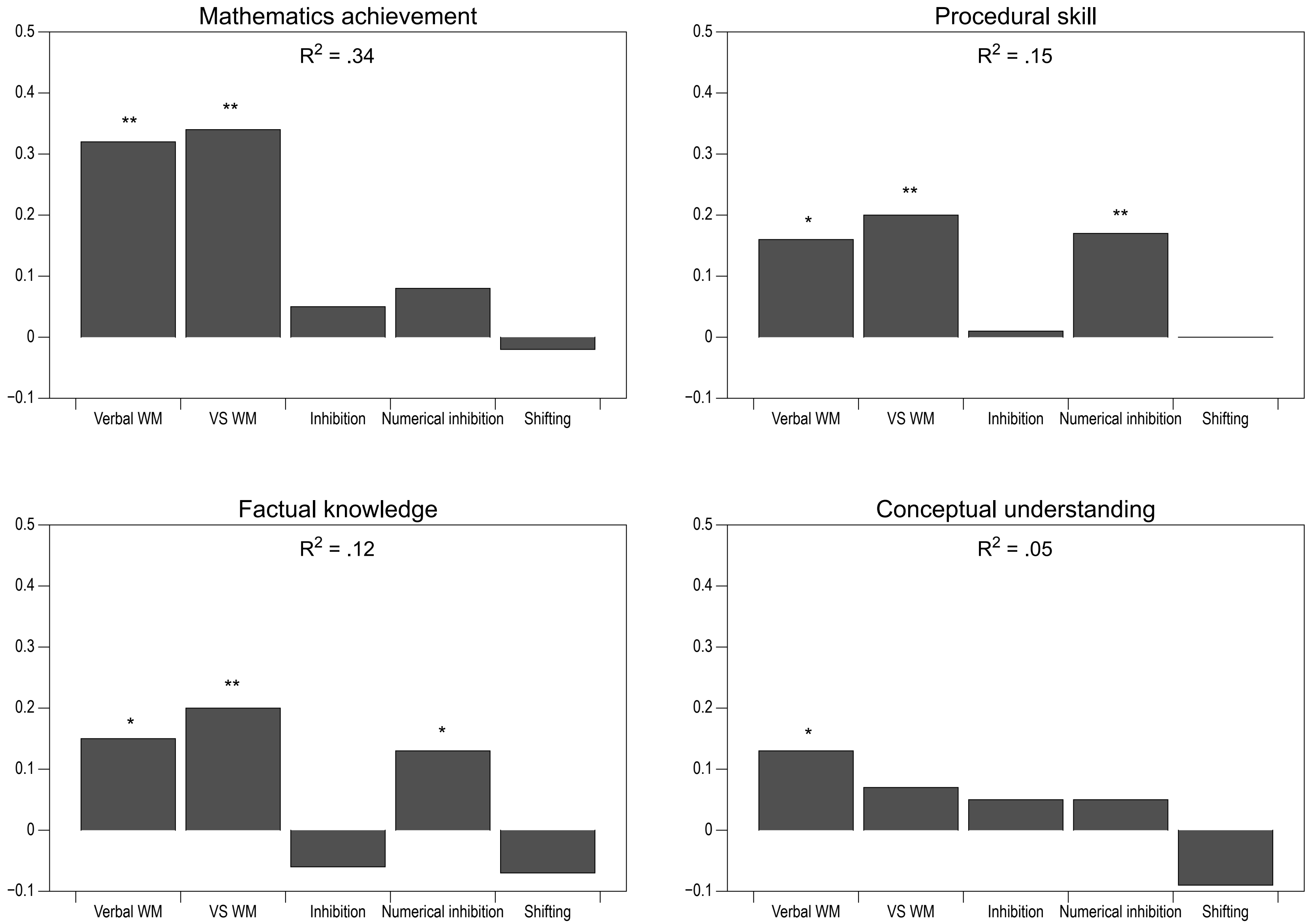
Mathematics achievement
R2 = .34
**
**
Procedural skill
R2 = .15
*
**
**
Factual knowledge
R2 = .12
*
**
*
Conceptual understanding
R2 = .05
*
0.5
0.4
0.3
0.2
0.1
0
−0.1
Verbal WM
VS WM
Inhibition
Numerical inhibition
Shifting